Analog Networks for High Heat-Transfer-Rate Measurements

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Analog networks for calculating automatically the rate of heat transfer between the model and the flowing gas from the surface temperature have been used by many investigators for several years.[1,2] However, the analogy between the heat flow inside the model and electric current flow inside a simple RC (resistance-capacitance)-type analog network holds only if the thermal properties of the model (thermal conductivity k, specific heat c_p, and density ρ) are constant. The properties of commonly used model materials such as pyrex, glass, or quartz can be considered constant only as long as the temperature variation does not exceed 10°C. Since in modern hypersonic test facilities the model surface temperature rise may even be as high as 100°C or more, using a simple analogy may introduce serious errors, and special precautions must be taken to avoid them.

The one-dimensional heat-conduction equation in the general case of a material with temperature-dependent properties is given by

$$\rho c_p(\theta)(\partial\theta/\partial t) = (\partial/\partial x)[k(\theta)(\partial\theta/\partial x)] \quad (1)$$

where θ = temperature, t = time, and x = spatial coordinate. This equation can be reduced to the form

$$\partial g/\partial t = \alpha(\partial^2 g/\partial x^2) \quad (2)$$

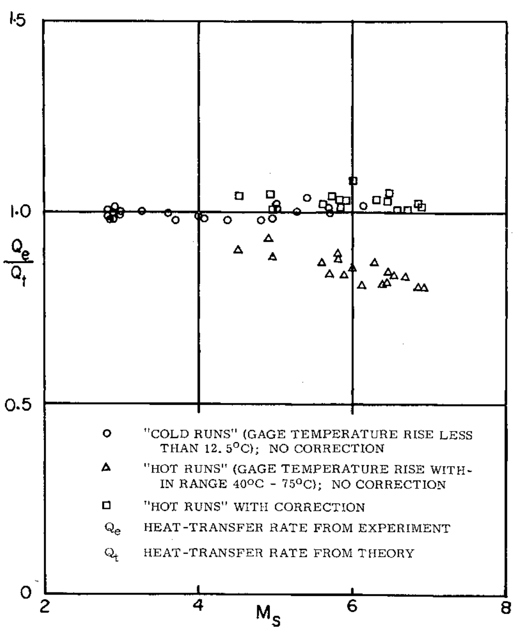

Fig. 1 Ratio of stagnation-point heat-transfer rates measured in the UTIAS 4- × 7-in. hypersonic shock tube and those calculated from the Fay and Riddell theory.

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(where $\alpha = k/\rho c_p$) by means of the transformation[3]

$$g = \int_0^\theta k(\theta')d\theta' \quad (3)$$

It has been found[3] that, in all practical cases, in the temperature range up to about 150°C, the variation of α can be neglected. It will be assumed then for simplicity that α = const; this assumption makes Eq. (2) linear.

In the case of an ideal material with constant properties ($k = k_0$ = const, c_p = const, and ρ = const), Eq. (3) reduces to

$$g = g_{\text{ideal}} = k_0\theta_{\text{ideal}} \quad (3a)$$

whereas Eq. (2) remains unchanged (i.e., it is identical with the equation for a material with variable properties and constant α).

It follows then that, for the same initial and boundary conditions, the function $g = g(x,t)$ is identical for materials with constant and variable properties, provided that α remains constant and in both cases has the same value.

From Eqs. (3) and (3a) it follows that

$$\theta_{\text{ideal}} = \frac{1}{k_0}\int_0^\theta k(\theta')d\theta' \quad (4)$$

Assuming k to be a linear function of temperature, i.e.,

$$k = k_0(1 + b\theta) \quad (5)$$

(validity of such an assumption follows from experiment[4]), Eq. (4) can be reduced to

$$\theta_{\text{ideal}} = \theta + (b/2)\theta^2 \quad (6)$$

The thin-film resistance gage, used in connection with the analog network, gives an output voltage u_g proportional to the model surface temperature θ_w, where $\theta_w = [\theta(x,t)]_{x=0}$. It follows then from Eq. (6) that

$$u_{g\ \text{ideal}} = u_g + \epsilon u_g^2 \quad (7)$$

where ϵ is a constant specified by the value b and the properties of the thin film, and $u_{g\ \text{ideal}}$ is an output from the gage mounted on an ideal material with constant properties.

The preceding relation constitutes the basis for the present modification of the analog network. If an electronic circuit, capable of performing a calculation according to Eq. (7) is connected between the gage and an ordinary analog network, then, although the output signal from the gage is affected by the variation of the thermal properties of the material, the input signal into the analog is identical with that for the material with constant properties, and the analog can furnish the correct heat-transfer rate.

An analog network with a correction circuit of the type described previously has been designed and built at the Institute for Aerospace Studies (see Ref. 4 for a detailed description). A series of experimental tests has been performed in the University of Toronto Institute for Aerospace Studies (UTIAS) 4- × 7-in. hypersonic shock tube. During these tests, two networks (with and without the correction circuit) were connected to the same thin-film gage, mounted on Pyrex 7740 glass along the stagnation line of a circular cylinder, and placed perpendicular to the flow. The results of the tests indicate that: 1) no correction is necessary if the rise of the model surface temperature during the experiment does not exceed 10°C, which is in agreement with the foregoing statement; 2) for surface temperature increments in the range from 40° to 75°C the analog network without correction gives heat-transfer rates 10 to 20% below the expected values; and 3) application of the correction circuit in the same range of the temperature increments reduces the average discrepancy between the heat-transfer rates expected and measured to about 3.5% only.

Figure 1 gives the ratios of the heat-transfer rates measured and calculated from the Fay and Riddell theory,[5] plotted in terms of the shock Mach number. In this figure the circles correspond to the cases of the model surface temperature rise equal to or less than 10°C without correction, the triangles refer to cases of temperature increments in the range of 40° to 75°C without correction, and the squares refer to the same cases with correction.

It is evident from the previous discussion that, for temperature increments up to 75°C, the proposed correction circuit works very well. It is expected that it will also work satisfactorily for temperature increments up to 150°C or more.

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Velocity Defect Laws for Transpired Turbulent Boundary Layers

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In earlier articles[4, 5] the present authors have proposed a "velocity defect law" for the transpired turbulent boundary layer with $dp/dx = 0$:

$$(U_1 - U)/U_\tau{}^* = f(y/\delta) \tag{1}$$

$U_\tau{}^*$ is the friction velocity based upon the shear stress at the inner edge of the outer flow.

Recently, Stevenson[7, 8] has proposed a "law of the wall"

$$\frac{2U_\tau}{v_0}\left\{\left(\frac{v_0 U}{U_\tau{}^2} + 1\right)^{1/2} - 1\right\} = \frac{1}{K}\ln\frac{yU_\tau}{\nu} + C \tag{2}$$

and a "velocity defect law"

$$\frac{2U_\tau}{v_0}\left\{\left(\frac{v_0 U_1}{U_\tau{}^2} + 1\right)^{1/2} - \left(\frac{v_0 U}{U_\tau{}^2} + 1\right)^{1/2}\right\} = F\left(\frac{y}{\delta}\right) \tag{3}$$

U_τ is the friction velocity based upon the wall shear stress. Stevenson showed that his experimental data, measured for axisymmetric flow with $dp/dx = 0$, v_0 = const, followed Eqs. (2) and (3). He also found that the data of Mickley and Davis,[3] which was for a plane flow, fit his relations, although the plane-flow results differed moderately from the axisymmetric flow data. The question that immediately arises is: Are the two defect laws represented by Eqs. (1) and (3) compatible?

In Figs. 1 and 2, new data[2] obtained in the authors' laboratory with plane flow, $dp/dx = 0$, $U_1 = 25$ fps, and $v_0/U_1 = 0.00266/x^{0.2}$ (x in inches) are plotted in the form of Eq. (1) (Fig. 1) and Eq. (3) (Fig. 2). Universal curves are obtained for both correlations. The data of Ref. 3 for $dp/dx = 0$, v_0 = const were also found to fall on the curves of Figs. 1 and 2. As a further check, Eqs. (2) and (3) were used in conjunction with the boundary-layer equations to calculate local shear stress as a function of y/δ for arbitrary positive values of v_0/U_1. The shear-stress profiles exhibited the expected broad maximum near $y/\delta = 0.1$, and $U_\tau{}^* = (\tau_{max}/\rho)^{1/2}$ was used to represent the friction velocity at the inner edge of the outer flow. A smooth curve through the data of Fig. 2 and representing Eq. (3) could then be converted to the form of Eq. (1) for arbitrary values of v_0/U_1. Up to the largest value of v_0/U_1 tested ($v_0/U_1 = 0.005$), a smooth curve through the data of Fig. 2 was transformed to a smooth curve through the data of Fig. 1.

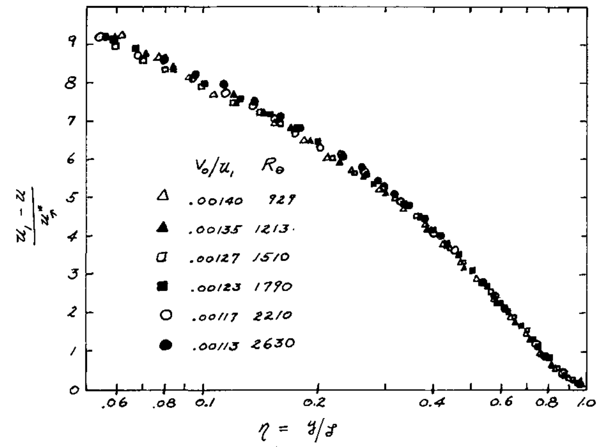

Fig. 1 Experimental confirmation of the $U_\tau{}^*$ form of the velocity defect relation, $dp/dx = 0$, variable v_0.

In Fig. 3, a relation between $U_\tau{}^*$ and U_τ is shown. This relation is based upon the authors' data, previous data,[1, 6] and calculations of $U_\tau{}^*$ made using Eqs. (2) and (3) as discussed previously. For $dp/dx = 0$, the ratio of $U_\tau{}^*/U_\tau$ is determined by local conditions at the wall with $v_0U_1/U_\tau{}^2$ as the governing parameter. From this result it follows that a defect law of the form

$$(U_1 - U)/U_\tau = \phi(y/\delta) \tag{4}$$

will be followed when $v_0U_1/U_\tau{}^2$ is constant, because then $U_\tau{}^*/U_\tau$ will be constant.

It is concluded that, for transpired turbulent boundary layers with $dp/dx = 0$ and v_0 varying smoothly with x, the

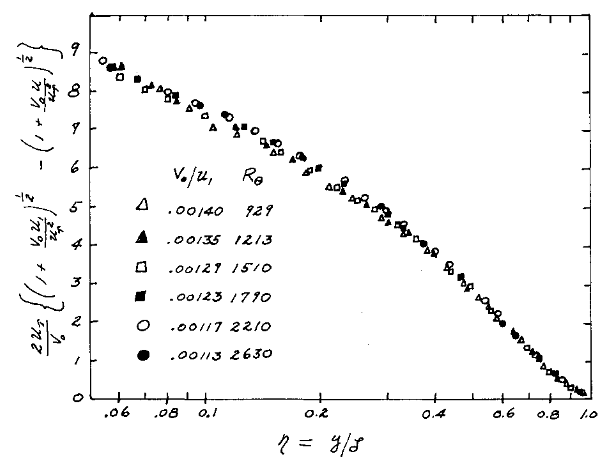

Fig. 2 Experimental confirmation of Stevenson's form of the velocity defect relation, $dp/dx = 0$, variable v_0.

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